

# IEEE Standard for Identity-Based Cryptographic Techniques using Pairings

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# IEEE Standard for Identity-Based Cryptographic Techniques using Pairings

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**IEEE Computer Society**

Approved 22 August 2013

**IEEE-SA Standards Board**

**Abstract:** Common identity-based public-key cryptographic techniques that use pairings, including mathematical primitives for secret value (key) derivation, public-key encryption, and digital signatures, as well as cryptographic schemes based on those primitives are specified in this standard. Also, related cryptographic parameters, public keys and private keys, are specified. The purpose of this standard is to provide a reference for specifications of a variety of techniques from which applications may select.

**Keywords:** encryption, identity-based encryption, IEEE 1363.3™, pairing-based cryptography, pairing-based encryption, public-key cryptography

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Tom Wu  
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Keith Chow  
Andy Dancer  
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## Introduction

This introduction is not part of IEEE Std 1363.3-2013, IEEE Standard for Identity-Based Cryptographic Techniques using Pairings.

This standard describes eight identity-based cryptographic schemes that use pairings in their implementation. The schemes include approaches to encryption, digital signatures, signcryption, and key exchanges. These schemes may be used to encrypt both stored data as well as data in transit. An underlying mathematical operation called a “pairing” is a common element of these schemes, and the standard describes algorithms for calculating pairings and gives parameters suitable for implementing the specified schemes at industry-standard security levels (NIST SP 800-57 [B125]).<sup>a</sup>

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<sup>a</sup> The numbers in brackets correspond to those of the bibliography in Annex F.

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## 1. Overview

### 1.1 Scope

This document specifies identity-based cryptographic schemes based on the bilinear mappings over elliptic curves known as pairings. Specific techniques include algorithms to compute the pairings and specification of recommended elliptic curves and curve parameters over which the pairings are defined. The class of computer and communications systems is not restricted.

### 1.2 Purpose

The proliferation of electronic communication and the Internet brings with it the need for privacy and data protection. Public-key cryptography offers fundamental technology addressing this need. Many alternative public-key techniques have been proposed, each with its own benefits. IEEE Std 1363<sup>TM</sup>-2000<sup>1</sup> and IEEE Std 1363a<sup>TM</sup>-2004 have produced a comprehensive reference defining a range of common public-key techniques covering key agreement, public-key encryption, and digital signatures from several families, namely the discrete logarithm, integer factorization, and elliptic curve families. This document will specify identity-based cryptographic techniques based on pairings. These offer advantages over classic public-key

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<sup>1</sup> Information on references can be found in Clause 2.