



IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices

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IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices

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Microprocessors and Microcomputers Committee
of the
IEEE Computer Society

Approved 10 December 2008
IEEE-SA Standards Board

Abstract: Specifications of common public key cryptographic techniques based on hard problems over lattices supplemental to those considered in IEEE Std 1363-2000 and IEEE Std 1363a-2004, including mathematical primitives for secret value (key) derivation, public key encryption, identification and digital signatures, and cryptographic schemes based on those primitives are provided. Also presented are specifications of related cryptographic parameters, public keys, and private keys. Class of computer and communications systems is not restricted.

Keywords: encryption, lattice-based cryptography, public key cryptography

The Institute of Electrical and Electronics Engineers, Inc.
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Introduction

This introduction is not part of IEEE Std 1363.1-2008, IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices.

The IEEE P1363™ project started as the “Standard for Rivest-Shamir-Adleman, Diffie-Hellman, and Related Public Key Cryptography” with its first meeting in January 1994, following a strategic initiative by the Microprocessor Standards Committee to develop standards for cryptography. Over the next eight years, the working group produced a broad standard reflecting the state of the art in public key cryptography, including techniques from three major families of hard problems. In addition, the working group drafted an addendum that provides additional techniques from those three major families. A more thorough history of the IEEE P1363 working group and its contributions beyond IEEE Std 1363™-2000 are given in the Introduction to IEEE Std 1363-2000.

At the same time, new cryptographic research was producing additional families of cryptographic techniques such as the family of techniques based on hard problems over lattices. These techniques enjoy operating characteristics that make them attractive in certain implementation environments, and thus they have received considerable scrutiny and deployment.

As a result, the working group proposed a new project to standardize techniques from this family. This project was approved by the Microprocessors and Microcomputers Standards Committee, and this current standard is the result of this project.

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Don Johnson, Vice Chair

Matthew Ball
Xavier Boyen
Mike Brenner
Daniel Brown
Mark Chimley

Andy Dancer
David Jablon
Satoru Kanno
David Kravitz
Michael Markowitz
Luther Martin

Jim Randall
Roger Schlafly
Ari Singer
Terence Spies
Yongge Wang

The following members of the individual balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

Ed Addario
Butch Anton
Matthew Ball
H. Stephen Berger
Martin J. Bishop
Juan Carreon
Keith Chow
Kevin Coggins
Geoffrey Darnton
James Davis
Thomas Dineen
Andrew Fieldsend

Michael Geipel
Randall Groves
Werner Hoelzl
Atsushi Ito
Mark Jaeger
Susan Land
David J. Leciston
Daniel Lindberg
Edward McCall
Avygdor Moise
Michael S. Newman
Ulrich Pohl

Robert Robinson
Randall Safier
Bartien Sayogo
Thomas Starai
Walter Struppler
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Vincent Tume
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David J. Law
Glenn Parsons
Ronald E. Petersen

Chuck Powers
Narayanan Ramachandran
Jon Walter Rosdahl
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Anne-Marie Sahazzia
Malcolm V. Thaden
Howard L. Wolfman
Don Wright

*Member Emeritus

Also included are the following nonvoting IEEE-SA Standards Board liaisons:

Satish K. Aggarwal, *NRC Representative*
Michael Janezic, *NIST Representative*

Don Messina
IEEE Standards Program Manager, Document Development

Malia Zaman
IEEE Standards Program Manager, Technical Program Development

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1. Overview

1.1 Scope

This standard provides specifications of common public key cryptographic techniques based on hard problems over lattices supplemental to those considered in IEEE Std 1363TM-2000 [B47]¹ and IEEE Std 1363aTM-2004 [B48], including mathematical primitives for secret value (key) derivation, public key encryption, identification and digital signatures, and cryptographic schemes based on those primitives. Specifications of related cryptographic parameters, public keys, and private keys are also presented. Class of computer and communications systems is not restricted.

1.2 Purpose

The transition from paper to electronic media brings with it the need for electronic privacy and authenticity. Public key cryptography offers fundamental technology addressing this need. Many alternative public key techniques have been proposed, each with its own benefits. IEEE Std 1363-2000 [B47] and IEEE Std 1363a-2004 [B48] have produced a comprehensive reference defining a range of common public key

¹ The numbers in brackets correspond to those of the bibliography in Annex B.