

# The Role of Uncertainty Quantification in Verification and Validation of Computational Solid Mechanics Models

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AN INTERNATIONAL STANDARD



The American Society of  
Mechanical Engineers

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# FOREWORD

Verification, validation, and uncertainty quantification (VVUQ) of engineering simulation models is currently of great interest to government, industry, and academia, all of which rely on computational modeling in the design and analysis of engineered structures. This has challenged government, industry, and academia to develop credible models for design and performance evaluation via computational simulation. In response, The American Society of Mechanical Engineers (ASME) formed a subcommittee on verification and validation (V&V) in computational solid mechanics (CSM), known as ASME V&V 10. The V&V 10 Subcommittee was redesignated as VVUQ 10 in 2021 since both V&V and uncertainty quantification (UQ) have been specifically identified as critical technologies needed for the advancement of computational mechanics.

The growing reliance on CSM models for decision-making in government, industry, and academia demands that greater attention be given to the quantification of uncertainties associated with these models. UQ is a critical component in the evaluation and communication of both computational and experimental results in the process of reporting simulation results. UQ is foundational in the development and assessment of CSM models and their predictive capability.

ASME V&V 10-2006 (ref. [1]), Guide for Verification and Validation of Computational Solid Mechanics Models, was recently revised to ASME V&V 10-2019 (ref. [2]), Standard for Verification and Validation of Computational Solid Mechanics Models. Both editions address an important need for a common language and process definition for VVUQ at the level appropriate for CSM model developers as well as their managers and customers. ASME V&V 10 is intended as an overview standard to be followed by detailed publications covering select topics and applications. ASME V&V 10.1 (ref. [3]) provides examples to illustrate many key VVUQ concepts. The overall purpose of this Standard is to expand on UQ, emphasized in ASME V&V 10.

This Standard describes the role of UQ in modeling/simulation and experimentation. UQ in modeling and simulation includes consideration of model form uncertainties, numerical solution uncertainties, model input uncertainties, and uncertainties in model-basis data. In addition, propagation of uncertainties is an integral part of UQ in modeling and simulation. UQ plays an important role in experimentation; therefore, key considerations in planning validation experiments are discussed, since these experiments are specifically planned and performed to assess the predictive capability of a computational model. A brief discussion of UQ in hierarchical CSM models is provided, as well as an overview of the role of UQ in revisions to either the computational model or the validation experiment.

This Standard also addresses the role of UQ in model validation assessment, illustrated by several examples considering different validation metrics that include uncertainties. Because validation metrics incorporate uncertainties in both experimental measurements and model simulations, some consideration of uncertainties is required in establishing the corresponding validation requirements.

This Standard is available for public review on a continuing basis. This provides an opportunity for additional public review input from industry, academic/regulatory agencies, and the public-at-large.

ASME VVUQ 10.2-2021 was approved by the VVUQ Standards Committee on August 27, 2021, and was approved and adopted by the American National Standards Institute on December 13, 2021.

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This Standard is always open for comment, and the Committee welcomes proposals for revisions. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

**Proposing a Case.** Cases may be issued to provide alternative rules when justified, to permit early implementation of an approved revision when the need is urgent, or to provide rules not covered by existing provisions. Cases are effective immediately upon ASME approval and shall be posted on the ASME Committee web page.

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Requests for interpretation should preferably be submitted through the online Interpretation Submittal Form. The form is accessible at <http://go.asme.org/InterpretationRequest>. Upon submittal of the form, the Inquirer will receive an automatic e-mail confirming receipt.

If the Inquirer is unable to use the online form, he/she may mail the request to the Secretary of the VVUQ Standards Committee at the above address. The request for an interpretation should be clear and unambiguous. It is further recommended that the Inquirer submit his/her request in the following format:

Subject: Cite the applicable paragraph number(s) and the topic of the inquiry in one or two words.  
Edition: Cite the applicable edition of the Standard for which the interpretation is being requested.  
Question: Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. Please provide a condensed and precise question, composed in such a way that a "yes" or "no" reply is acceptable.  
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Background Information: Provide the Committee with any background information that will assist the Committee in understanding the inquiry. The Inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in the format described above may be rewritten in the appropriate format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

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# INTRODUCTION

Model verification, validation, and uncertainty quantification (VVUQ) are the primary methods for quantifying and building credibility in mathematical/computational models. Verification is the process of determining that a computational model accurately represents the underlying mathematical model and its solution. Validation is the process of determining the degree to which a model is an accurate representation of the validation experiments from the perspective of the intended uses of the model. Both verification and validation accumulate evidence of model correctness and accuracy for a specific application of interest. Model VVUQ\* cannot prove that the model is correct and accurate for all possible scenarios; instead, VVUQ accumulates evidence of whether the model is sufficiently accurate for its intended uses. The expected outcome of the VVUQ process is to inform the decision-maker of the credibility of the model for the intended uses of the model.

Uncertainty quantification (UQ) in the context of VVUQ is the mathematical assessment of uncertainties in model simulation results and experimental results. Therefore, the goal is to quantify the uncertainties in both simulation and experimental results so that the model accuracy can be assessed, and the predictive capability of the model can be established quantitatively. As described in this Standard, UQ is also important in other related activities, including model development, parameter estimation, model calibration, experimental design, and sensitivity analysis.

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\*For the purposes of this Standard, “model VVUQ” will be referred to as “VVUQ.”

# THE ROLE OF UNCERTAINTY QUANTIFICATION IN VERIFICATION AND VALIDATION OF COMPUTATIONAL SOLID MECHANICS MODELS

## 1 PURPOSE AND SCOPE

### 1.1 Purpose and Motivation

The purpose of this Standard is to expand upon the important role of uncertainty quantification (UQ) in verification, validation, and uncertainty quantification (VVUQ), as outlined in [Figure 1.1-1](#). UQ plays an important part in each of the “Modeling and Simulation” and “Physical Experimentation” branches illustrated in the figure, ultimately quantifying the uncertainties in the “Simulation Results” and “Experimental Results” generating the “Simulation Outputs” and “Experimental Outputs.” A detailed description of this figure is provided in ASME V&V 10-2019.

Consistent with the purpose of ASME V&V 10-2019, the motivation for developing ASME VVUQ 10.2 is the need for a common language and process of UQ in computational solid mechanics (CSM), particularly as it may relate to how model developers perform UQ as well as how they subsequently communicate results, conclusions, and recommendations to a decision-maker. A decision-maker may be any individual or representative entity, such as a review panel, deemed responsible for determining if a model is acceptable for its intended uses. The decision-maker may also be a customer relying on model predictions to inform a decision.

### 1.2 Objectives and Scope

- 1.2.1 Objectives.** The objectives of this Standard are to
- (a) define and clarify the role of UQ as part of the VVUQ process
  - (b) provide guidance for the use of UQ in VVUQ activities
  - (c) acknowledge the importance of UQ in decision-making

- 1.2.2 Scope.** The scope of this Standard includes the following:
- (a) sources and types of uncertainty and how they can be treated in the VVUQ process ([section 2](#))
  - (b) quantification and propagation of uncertainties ([section 3](#))
  - (c) uncertainties in validation experiments ([section 4](#))
  - (d) uncertainties in model validation assessment ([section 5](#))
  - (e) revisions to the model and experiments ([section 6](#))
  - (f) uncertainties in hierarchical models ([section 7](#))

## 2 BACKGROUND AND DEFINITIONS

### 2.1 Mathematical Models

Models are idealized representations of the physical phenomena of interest. This Standard distinguishes between two types of mathematical models: empirical models and physics-based models. Examples of both types of mathematical models are provided in [para. 2.2](#). These are defined as follows:

*empirical model:* a mathematical model whose functional framework is based primarily on observation and experiment. In CSM, empirical models are typically expressed in closed-form algebraic relations, which may still encompass some level of our conceptual or theoretical understanding of the physical phenomenon of interest. Empirical models may be simple statistical models (e.g., regression analysis) or more complex (e.g., based on machine learning or generalized polynomial expansions). Experimental data used to develop the empirical model are hereafter referred to as *model-basis data*. In the CSM community, this type of data is also referred to as *calibration data*.