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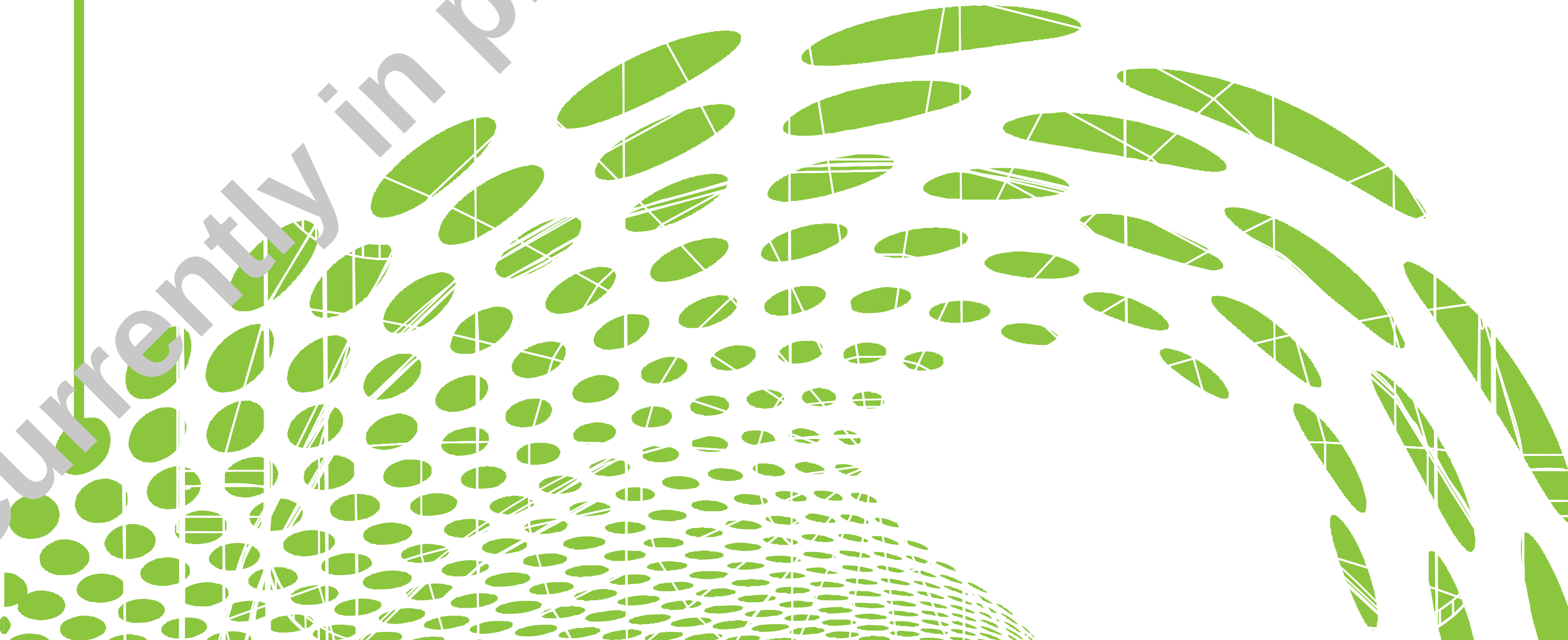
STP-PT-082

STRESS INTENSITY K FACTORS FOR EXTERNAL SURFACE CRACKS IN THICK-WALLED CYLINDER VESSELS



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STRESS INTENSITY K FACTORS FOR EXTERNAL SURFACE CRACKS IN THICK-WALLED CYLINDER VESSELS

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ASME STANDARDS
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FOREWORD

Established in 1880, ASME is a professional not-for-profit organization with more than 135,000 members and volunteers promoting the art, science and practice of mechanical and multidisciplinary engineering and allied sciences. ASME develops codes and standards that enhance public safety, and provides lifelong learning and technical exchange opportunities benefiting the engineering and technology community. Visit www.asme.org for more information.

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ABSTRACT

This report describes the analysis methods and results for external surface crack stress intensity K solutions in thick-walled cylinders. The 1040 cases include a range of geometry ratios and crack size ratios for external axial and external circumferential surface cracks, axial full-width cracks, and 360° circumferential cracks. These K solutions extend the K factor solutions available in the 2007 API 579-1/ASME FFS-1 Annex C tables. The results are reported as non-dimensional geometry factors that are tabulated in the appendices along with plots of all the result cases.

1 INTRODUCTION

ASME ST-LLC's request-for-proposal RFP-16-03 (project STIN-0151) sought the calculation of stress intensity K factors for external surface cracks in thick-walled cylinder vessels. The 1040 crack analysis cases for this project extend the K factor solutions available in the API 579-1/ASME FFS-1 [1] Annex C tables. This project supplements the results from ASME ST-LLC's project STIN-0130 for the internal crack solutions as identified in ASME ST-LLC's publication STP-PT-072 [2].

The analysis method uses our FEACrack™ [3] software to generate the three-dimensional ("3D") crack meshes, described below. Quest Integrity developed the FEACrack™ software, a commercial product for 3D crack mesh generation and analysis, and has continuously verified its robustness and suitability for analyses such as in the current work. This software was originally released in 1998, and was used to create most of the K factor solutions in Annex C of API 579 [1][4][5]. FEACrack™ creates complete and ready-to-run Abaqus™ [6] input files, including the syntax to define the J-integral calculation, to allow efficient analysis of many crack cases.

The finite element analysis cases are run for each crack mesh using the Abaqus™ solver. Abaqus™ also provides the crack front J-integral calculation at the crack front nodes. FEACrack™ provides automated post processing to help inspect the mesh and crack front J-integral results and tabulate the stress intensity solution factors. FEACrack™ automatically computes the stress intensity K factor from the J-integral using the elastic material properties, and examines the J-integral path dependence to indicate any issues with a result.

The stress intensity results are reported as non-dimensional geometry G factor values, described in Section 1.2. The result values are tabulated in the appendices. Plots of the results for all the cases examined and for result trends are also shown in the appendices. Each appendix begins with a description of the values or plots within the particular appendix.

1.1 Analysis Cases

The crack analysis cases described in the ASME ST-LLC scope of work use the geometry ratio Y = OD/ID and the a/l crack aspect ratio, where OD is the cylinder outside diameter, ID is the cylinder inside diameter, a is the crack depth, and l is the total semi-elliptical crack length. The crack depth ratio a/t describes the crack depths examined, where t is the cylinder wall thickness.

The Y and a/l ratios are related to the t/Ri and a/c ratios used in the API 579 Annex C tables, where Ri is the cylinder inside radius and c is the half semi-elliptical surface crack length: t/Ri = Y-1, a/l = (a/c)/2, 2c = l. The same a/l and a/c ratio values as the Annex C solutions were used so that the new stress intensity solutions can be easily added to the existing solution tables. Both sets of ratios are given in Table 1 through Table 3 below. The Y = 2 ratio (t/Ri = 1) overlaps the current solutions so that the new results can be compared to show continuity of values. The new solutions extend the Y ratio to 4 (t/Ri = 3) for the thickest cylinder case examined.

Generic values for geometry and loads are used to create the crack meshes, since the final results are given as the non-dimensional geometry G factors.

Table 1: Y and t/Ri Ratios to Set the Cylinder Thickness, t

Case	Y=OD/ID	t/Ri
1	2	1
2	2.5	1.5
3	3	2
4	3.5	2.5
5	4	3

Table 2: a/l and a/c Ratios to Set the Crack Length, l=2c

Case	a/l=a/2c	a/c
1	0.01563	0.03125
2	0.03125	0.0625
3	0.0625	0.125
4	0.125	0.25
5	0.25	0.5
6	0.5	1
7	1	2
8	360-deg or full width	

Table 3: a/t Ratios to Set the Crack Depth, a

Case	a/t
1	0.2
2	0.4
3	0.6
4	0.8

Some of the a/l crack length ratios give circumferential crack lengths that are longer than the thick-walled cylinder outside circumference, so these cases are omitted. Table 4 summarizes the valid circumferential surface crack cases, including the 360-degree crack cases that provide a bounding case for the circumferential crack solutions.

Table 4: External Circumferential Crack Valid Cases

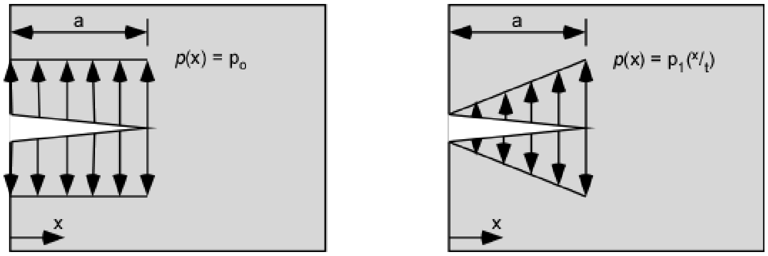
Y=OD/ID =	2							
	a/l=a/2c							
a/t	360-deg	0.015625	0.03125	0.0625	0.125	0.25	0.5	1
0.2	ok	too long	ok	ok	ok	ok	ok	ok
0.4	ok	too long	too long	ok	ok	ok	ok	ok
0.6	ok	too long	too long	ok	ok	ok	ok	ok
0.8	ok	too long	too long	too long	ok	ok	ok	ok
Y=OD/ID =	2.5							
	a/l=a/2c							
a/t	360-deg	0.015625	0.03125	0.0625	0.125	0.25	0.5	1
0.2	ok	too long	ok	ok	ok	ok	ok	ok
0.4	ok	too long	too long	ok	ok	ok	ok	ok
0.6	ok	too long	too long	ok	ok	ok	ok	ok
0.8	ok	too long	too long	too long	ok	ok	ok	ok
Y=OD/ID =	3							
	a/l=a/2c							
a/t	360-deg	0.015625	0.03125	0.0625	0.125	0.25	0.5	1
0.2	ok	too long	ok	ok	ok	ok	ok	ok
0.4	ok	too long	too long	ok	ok	ok	ok	ok
0.6	ok	too long	too long	too long	ok	ok	ok	ok
0.8	ok	too long	too long	too long	ok	ok	ok	ok
Y=OD/ID =	3.5							
	a/l=a/2c							
a/t	360-deg	0.015625	0.03125	0.0625	0.125	0.25	0.5	1
0.2	ok	too long	ok	ok	ok	ok	ok	ok
0.4	ok	too long	too long	ok	ok	ok	ok	ok
0.6	ok	too long	too long	too long	ok	ok	ok	ok
0.8	ok	too long	too long	too long	ok	ok	ok	ok
Y=OD/ID =	4							
	a/l=a/2c							
a/t	360-deg	0.015625	0.03125	0.0625	0.125	0.25	0.5	1
0.2	ok	too long	ok	ok	ok	ok	ok	ok
0.4	ok	too long	too long	ok	ok	ok	ok	ok
0.6	ok	too long	too long	too long	ok	ok	ok	ok
0.8	ok	too long	too long	too long	ok	ok	ok	ok

1.2 Geometry Factors

The stress intensity K factors are given in the API 579 [1] Annex C tables as non-dimensional geometry factors: G_0 and G_1 for the surface cracks, where G_0 is the uniform crack face pressure solution and G_1 is the linear crack face pressure solution. Geometry factors G_0 through G_4 are given for the axial full-width and circumferential 360-degree partial-depth cracks. Uniform and linear crack face pressure distributions are applied to the surface crack meshes to obtain the geometry factors for these two load cases. The full-width and 360-degree cracks have uniform, linear, quadratic, cubic, and quartic (fourth order) crack face pressure distributions applied to obtain their geometry factor solutions.

Diagrams of the uniform and linear crack face pressure distributions are shown in Figure 1-1 [7], and the general form of the stress intensity K equations with the geometry factor G are shown below each diagram. The linear crack face pressure is zero at the free surface and increases toward the crack depth. Likewise, the quadratic, cubic, and quartic crack face pressures increase from zero at the free surface to maximum at the crack depth.

Figure 1-1: Crack Face Pressure Distributions



Uniform Crack Face Pressure

Linear Crack Face Pressure

$$K_I = p_o G_o \sqrt{\frac{\pi a}{Q}} \qquad (1)$$

$$K_I = p_1 G_1 \sqrt{\frac{\pi a}{Q}} \qquad (2)$$

The circumferential surface cracks also have two global bending K solutions in API 579 [1] Table C.13 that provide the G_5 (in-plane bending) and G_6 (out-of-plane bending) geometry factor solutions. Since the bending load cases can put part of the surface crack into compression and would cause crack face closure, superposition is used to apply a combined bending plus axial load, so that the entire crack front is in tension for the finite element analysis solution. Then, subtracting the axial load solution from the combined load solution obtains the bending only solution for each case, which may be negative due to compressive stress. To keep the crack front in tension the axial load is six times the bending load to avoid crack closure in some of the larger crack size cases.

For example, the entire crack front is put into tension by applying a combined axial force and bending moment, which gives the total crack front stress intensity K_{total}. The axial force is applied by itself in another model to get the crack front stress intensity K_{axial}. The principle of superposition allows combining load cases in linear elastic analysis such that: $K_{total} = K_{bending} + K_{axial}$. The bending only stress intensity is obtained by subtracting the axial load case from the combined load case: $K_{bending} = K_{total} - K_{axial}$. The bending only K solutions can be negative so that when combined with other load cases in a crack assessment the correct sum of the K solutions from each loading component is obtained. In a crack assessment, a total negative K value after combining all loads indicates crack closure and K is set to zero.

The FEACrack™ post processor computes the crack front stress intensity K solutions from the Abaqus™ J-integral results and uses the applied loading to compute the non-dimensional geometry G factors. A sixth order polynomial is used to curve-fit the G solution along the crack front. The seven polynomial coefficients are tabulated for each crack case to give the same result format as in the API 579 [1] Annex C tables. The geometry factor polynomial, G, is given by the equations below [1] (equations C.91 and C.96).

$$G = A_0 + A_1\beta + A_2\beta^2 + A_3\beta^3 + A_4\beta^4 + A_5\beta^5 + A_6\beta^6 \qquad (3)$$

$$\beta = \frac{2\varphi}{\pi} \qquad (4)$$

where the crack front position angle, φ, varies from 0 at the crack tip (at the free surface) to $\pi/2$ at the deepest point of the surface crack. Likewise, the non-dimensional crack front position, β, varies from 0 at the crack tip to 1 at the crack depth. The polynomial curve fit coefficient values are given by the A_0 through A_6 values. The geometry factor and stress intensity solutions at the crack tip location are obtained by setting $\beta = 0$, which gives just the A_0 coefficient. The geometry factor and stress intensity solutions at the crack depth location are given by setting $\beta = 1$, which is the sum of the A_i coefficient values: $G(\beta=1) = \Sigma Ai$,

$i=0,6$. By reporting the polynomial coefficients, A_0 through A_6, the stress intensity solution is available along the entire crack front in addition to the crack tip and crack depth locations.

The axial full-width and circumferential 360° partial-depth cracks have a constant stress intensity value along the crack front, so only the A_0 coefficient is reported for the geometry factors for each crack face pressure load case.

The non-dimensional geometry factor solutions obtained for the uniform and linear surface crack face pressure load cases can be used to infer the solutions for other loading distributions by using the weight function method as described in the API 579 [1] Section C.14. The weight function method uses equations and weighting factors to combine the G_0 and G_1 solutions to obtain other load case solutions. For example, the non-uniform stress distribution due to internal pressure in a thick-walled cylinder could be used to obtain the corresponding G solution using the weight function method.