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# COMPARISON AND VALIDATION OF CREEP-BUCKLING ANALYSIS METHODS



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# COMPARISON AND VALIDATION OF CREEP BUCKLING ANALYSIS METHODS

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## FOREWORD

This document was developed under a research and development project which resulted from ASME Pressure Technology Codes & Standards (PTCS) committee requests to identify, prioritize, and address technology gaps in current or new PTCS Codes, Standards and Guidelines. This project is one of several included for ASME fiscal year 2008 sponsorship which are intended to establish and maintain the technical relevance of ASME codes & standards products. The specific project related to this document is project 07-11 (BPVC#5), entitled “Comparison and Validation of Creep-Buckling Analysis Methods.”

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## ABSTRACT

This report provides comparisons of creep-buckling calculations and provides guidance on approximate methods which are feasible for design. This report includes a discussion of the various creep models, presents creep buckling analysis techniques, and provides several comparative example calculations.

The techniques discussed in this report include:

1. Baseline analysis. Finite element creep analysis with different creep models and full non-linear strain-displacement (geometrical) analysis.
2. Critical strain technique. Elastic buckling strain defines the creep buckling strain.
3. Tangent/secant modulus approaches. Combinations of tangent and secant moduli of the isochronous stress-strain curve are used in calculations that reduce to elastic buckling calculations in the elastic case.
4. Use of an isochronous stress-strain curve in a limit/instability analysis of the imperfect structure. An instability (buckling) analysis would be in principle the same as Technique 3 and should generate the same answer. Adding plastic collapse as a failure mode ensures that the yield strength of the structure is not exceeded. This analysis therefore reflects the failure modes which are covered by the baseline technique.

## 1 INTRODUCTION

This report provides comparisons between approximate and detailed creep-buckling calculations. The objective is to provide guidance on approximate methods which are feasible for design. This requires the efficient calculation of structural strength and time to (buckling) failure, so that calculation of margins between design and failure boundaries does not require multiple trial and error creep calculations. The definition of creep buckling is taken to be wide, including elastic and inelastic instability, bifurcation and acceleration of strain and deflection rates due to non-linear geometrical reduction in structural strength.

The techniques used in this report are:

1. Baseline analysis. Finite element creep analysis with different creep models and full non-linear strain-displacement (geometrical) analysis.
2. Critical strain technique. Elastic buckling strain defines the creep buckling strain. ([1], [2], [3])
3. Tangent/secant modulus approaches. Combinations of tangent and secant moduli of the isochronous stress-strain curve are used in calculations that reduce to elastic buckling calculations in the elastic case. ([4], [5], [6])
4. Use of an isochronous stress-strain curve in a limit/instability analysis of the imperfect structure. An instability (buckling) analysis would be in principle the same as technique 3, and should generate the same answer. Adding plastic collapse as a failure mode ensures that the yield strength of the structure is not exceeded. This analysis therefore reflects the failure modes which are covered by the baseline technique.

Techniques 2 and 3 do not have an explicit treatment of initial imperfection or out-of-roundness. For simple structures such as cylinders and spheres, Technique 1 requires an initial imperfection to give a reasonable result. With no defined initial imperfection it may or may not give a result, and if there was a result, it may or may not bear any resemblance to reality. Technique 4 requires the same initial imperfection as 1 to give a reasonable result.

The selection of the initial imperfection is simple for the cases considered in this report. It is the first elastic buckling mode shape with a defined magnitude. For more complex structures, it may be necessary to examine a number of possible imperfection mode shapes, and to base the strength prediction on the mode which gives the most conservative result. This is conveniently done by using a range of elastic buckling mode shapes, but other plausible or defined imperfection shapes can easily be used.

A 0.5 mm radial imperfection with 100 mm radius corresponds to the ASME definition of 1% maximum acceptable out-of-round. This and 0.1 mm imperfections are considered in this report.

Plasticity is not included. The cases to be analyzed will represent reasonable design conditions in terms of stress, temperature and life. Under these circumstances significant plasticity would not be expected for the simple structures in this report, unless it occurred due to severe distortions late in life. It would be difficult to load these structures so that initial yielding occurred which did not lead to instantaneous elastic-plastic buckling. In this case there is no difference between the technique 4 limit/instability analysis and the Technique 1 baseline analysis. However, plasticity may be readily included in all the analyses if necessary. There is no reason why isochronous stress-strain curves constructed from tests or from full elastic-creep-plasticity properties should present any difficulties over and above those in this report.

Inclusion of plasticity in the full inelastic analysis and in the three approximate methods is not expected to change the conclusions based on the creep models. The ability of the approximate