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# Report on Indirect Method to Obtain Stress- Strain Response of Fiber- Reinforced Concrete (FRC)

Reported by ACI Committee 544



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## Report on Indirect Method to Obtain Stress-Strain Response of Fiber-Reinforced Concrete (FRC)

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# Report on Indirect Method to Obtain Stress-Strain Response of Fiber-Reinforced Concrete (FRC)

Reported by ACI Committee 544

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*Development of proper design procedures for fiber-reinforced concrete (FRC) materials requires use of material tensile and compressive stress strains that reflect the contribution of fibers to the post-cracking behavior. While uniaxial tension tests provide the most fundamental material properties, conducting closed-loop tension tests are difficult to accomplish; therefore, methods based on indirect measurement of tensile properties using flexural tests are typically used.*

*This report presents the methodologies that are used for data reduction and presentation of the flexural test results in terms of an equivalent tensile stress-strain response for FRC materials. Existing methods for estimating uniaxial tensile stress-strain response of strain-softening and hardening FRCs from flexural beam-test data are introduced. Different approaches applied to beam tests based on elastic equivalent, curve fitting, or back-calculation of flexural data are introduced. These are divided into two general categories: elastic equivalent approach or inverse analysis method. In the elastic equivalent approach, a summary of available test methods by various code agencies are presented.*

*Using back-calculation methods, tools based on the finite element method and analytical closed-form solutions are presented. An approach is presented that uses closed-form moment-curvature relationships and obtains load-deflection responses for a beam of three- or four-point loading. The method is used to obtain equivalent parametric tensile stress and strain relationships for a variety of FRC materials. The methods are compared against the available residual strength and also elastically equivalent residual strengths obtained by different specimen geometries.*

*Results for a range of FRC materials studied show the back-calculated post-peak residual tensile strength is approximately 30 to 37 percent of the elastically equivalent flexural residual strength for specimens with different fiber types and volume fractions.*

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**Keywords:** fiber-reinforced concrete; inverse analysis; tensile stress-strain diagram.

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**CHAPTER 1—INTRODUCTION AND SCOPE****1.1—Introduction**

This report provides guidelines for obtaining uniaxial stress-strain curves of fiber-reinforced concrete (FRC) from

beam test data. In many FRC systems, the contribution of fibers is apparent after the concrete cracks and the fibers that bridge such cracks start to debond and pullout, thus resisting its opening and generating a force that transfers the loads across the crack. The magnitude of load carried by the fibers depends on the opening of the crack width; it can be normalized with respect to the cracked area and referred to as a residual strength. The role that fibers play in bridging a main tensile crack is therefore characterized as resisting crack opening, also referred to as bridging force and is represented as an average effective stress and described by a tensile stress-crack width relationship. A majority of FRC mixtures exhibit a distinct stress-crack width relationship that can also be integrated with the initial elastic response of the composite into a combined nonlinear stress-strain response.

The stress-crack width relationship can be represented as an equivalent stress-strain response by assuming a characteristic length parameter to smear the crack width into a nominal strain distribution. If mechanical tests that only focus on the tensile strength (ASTM C496/C496M) or flexural strength (ASTM C78/C78M) of the FRC are conducted, this contribution, herein referred to as residual strength, is either inaccurate, not reported, or reported in terms of parameters that may not be useful for design or analysis. Furthermore, accurate measurements of tension tests that capture the post-peak response are difficult to conduct; therefore, many agencies use flexural tests as an indication of tensile response.

While the tension test theoretically shows the true material behavior and the flexural test represents a structural response, a flexural test is often used as a means of property measurement. The difference between the tension and flexural test results of many FRC materials is that in a tension test, the post-peak tensile stress-crack width response does not influence the maximum load obtained by the member. In a flexural test, however, the maximum load can be directly related to the residual stress levels such that the overall behavior can be affected by the post-peak response.

An alternative method is to calculate post-cracking behavior using the experimental flexural results and reduce them into a set of material parameters that are in compliance with the model assumptions. This topic is the subject of this report. The testing methodologies are discussed in detail in committee reports such as ACI 544.3R. The present report addresses procedures to obtain an effective tensile stress-strain curve from the experimental results.

Many structural systems that use FRC, such as structural floors or indeterminate structures, can exhibit an increase in strength values in proportion to the residual strength, which is a direct contribution of fibers; however, this parameter is only measured in a qualitative way using flexural tests such as ASTM C1609/C1609M.

To develop and apply design procedures for FRC materials, simplified equations are needed to account for the fiber's contribution to the tensile response, especially after cracking has occurred. This report addresses methods to compute the stress transfer after cracking is initiated in a concrete section.